A Robust Hidden Markov Gauss Mixture Vector Quantizer for a Noisy Source

Kyungsu (Peter) Pyun, Member, IEEE, Johan Lim, and Robert. M. Gray, Fellow, IEEE

Abstract

Noise is ubiquitous in real life and changes image acquisition, communication, and processing characteristics in an uncontrolled manner. Especially, Gaussian noise and Salt and Pepper noise are prevalent in noisy communication channels, camera and scanner sensors, medical MRI images, and etc. It is not unusual for highly sophisticated image processing algorithms developed for clean images to malfunction when used on noisy images. For example, hidden Markov Gauss mixture models (HMGMM) have been shown to perform well in image segmentation applications, but they are quite sensitive to image noise. We propose a modified HMGMM procedure specifically designed to improve performance in the presence of noise. The key feature of the proposed procedure is the adjustment of covariance matrices in Gauss mixture vector quantizer codebooks to minimize an overall minimum discrimination information distortion (MDI). In adjusting covariance matrices, we expand or shrink their elements based on the noisy image. While most reported literature assumes a particular noise type, we propose universal framework without assuming particular noise characteristics. Without denoising the corrupted source, we apply our method directly to segmenting noisy sources. We apply the proposed procedure to the segmentation of aerial images with Salt and Pepper noise and with independent Gaussian noise, and we compare our results with those of the median filter restoration method and the blind deconvolution based method, respectively. We show that our procedure has better performance than image restoration based techniques and closely matches to the performance of HMGMM for clean images in terms of both visual segmentation results and error rate.

Index Terms

Signal and noise modeling, hidden Markov model, Gauss mixture model, Image segmentation, Image classification, Pattern recognition, Machine learning, Supervised learning, Covariance adjustment, Salt and Pepper noise, independent Gaussian noise

I. INTRODUCTION

We consider a statistical image segmentation problem based on quantization ideas when the images are noisy. Image segmentation extracts explicit information about content and allows human observers to understand images clearly by focusing on specific regions of interest. Hence, image segmentation is often used as an efficient initial procedure to simplify a sophisticated and complex algorithm [6], [27]. For example, if segmentation divides an image into several homogeneous regions, a compressor can quantize
the image more efficiently by focusing on each homogeneous region [9]. Classification can be considered as a form of compression, and vice versa, since assigning an image into a labelled class can be understood as a mapping from inputs into labels using an encoder. This viewpoint provides a clustering approach to the fitting of Gauss mixture models (GMM) to data and lossy compression algorithms based on Gauss mixture vector quantization (GMVQ), which can be used to classify different regions of interest using a minimum distortion rule [2], [27], [28]. The Gaussian probability density function (pdf) maximizes the differential entropy, which maximizes both the Shannon distortion-rate function and the entropy-constrained high rate optimal performance given the mean and covariance [14], [29]. This implies that GMVQ provides a robust approach to quantization of general sources with smooth density functions [14].

Although GMVQ has been found to be a good classifier, it is not necessarily a good segmenter since the latter requires smooth boundaries between classes. To this end, hidden Markov models (HMM) have often been used to model the coherence between neighborhood classes [23], [27]. In particular, Pyun et al. [27] introduced a hidden Markov Gauss mixture model (HMGMM) which incorporates the spatial coherence between classes into GMM. An HMGMM models the density of intra-block vectors using a GMVQ for each class, where the parameters of the GMVQ are determined using the generalized Lloyd algorithm with minimum discrimination information (MDI) distortion measure [2]. On the other hand, the HMGMM models the inter-block information using the simplest HMM, similar to the bond percolation model [15], [19]. To test a new image, the algorithm jointly segments the image and estimates the parameters of the HMM. Extensive experiments demonstrated that the HMGMM provides good segmentation results as measured by misclassification rate. When the images are corrupted by noise, however, the performance of HMGMM degrades.

Noise occurs naturally when images are transmitted through communication channels or acquired by image camera sensors and scanners, or magnetic resonance imaging (MRI) system [10], [20]. This paper focuses on coding a remote noisy source, in particular on the case of a source corrupted by acquisition noise. There are extensive studies dealing with communication noise incurred during transmission in the literature [8], [11], [30], [31]. With regard to the acquisition noise, Tramini et al. [34] proposed joint denoising and deblurring using multi-resolution approach to reduce the effects of acquisition noise, They applied their methods to satellite images. Wu et al. [36] used both modified boundary Markov random field (MRF) and a label MRF to segment complex boundaries when images are corrupted by noise, and they applied their methods to medical image segmentation problem. Ahmed et al. [1] modified an objective function of a standard fuzzy C-means algorithm and used neighborhood regularization to
segment MRI images under the influence of Salt and Pepper noise. All these papers, however, assume particular noise characteristics for their approach, whereas our approach does not make any assumption based on a specific noise model.

Robust signal processing techniques are methods that provide good results even when the model used for algorithm design and theory contains uncertainty regarding to information source behavior. A general example is the design of a signal processing system, such as a compression or classification system, based on a "clean" model of the original source; the system is considered robust if the resulting algorithm still perform reasonably well if applied to data that is corrupted by additive, signal-independent noise. More specific examples include speech recognition systems designed to operate in a noisy environment such as pilot speech in an aircraft, equipment operator speech on a factory floor, and mobile radio voice in traffic noise. Ephraim and Gray [7] provided an unified approach for clean and noisy sources and applied it to robust speech recognition. Their method assumed knowledge of the noise source and used minimum distortion encoding with respect to a modified distortion measure that incorporated the noise statistics. They showed that under suitable technical assumptions (autoregressive sources and Gaussian noise), a natural ad hoc approach of first finding an optimal nonlinear estimate of the original signal and then coding the estimate was approximately optimal in an overall sense. The approach unfortunately provided little insight for the problem of designing a robust code in the absence of statistical knowledge of the noise.

Noise can result a mismatch between a code book for template matching in a compression or classification system that is designed based on clean images and a noisy image. The GMVQ-based classifiers [27] rely only on the test data through mean vectors and covariance matrices of each sub-block. Hence, they are affected by the noise only through the mean vector and covariance estimates. If the mean of the noise is zero, then the mismatch is only through the covariance matrix estimate. In other words, noise provides biased covariance estimates, to which the HMGMM can have a sensitive response.

This paper proposes a robust hidden Markov Gauss mixture vector quantizer (HMGMMVQ) for encoding a noisy source. The basic HMGMVQ is designed for a clean source as in [27] based on the approach of Ephraim and Gray [7], but the covariance estimation stage for encoding new data is modified to account for the uncertainty in the estimation introduced by added noise. The modification is motivated by Stein’s covariance matrix estimator based methods [16], [22], [32]. Instead of restoring noisy images first and then segmenting the restored image, we segment the noisy images directly. Our method induces a new Gauss mixture vector quantizer for noisy images. This approach can be considered as a form of modifying
the trained GMVQ in the compressed domain, as will be seen. The algorithm has a single stage two
dimensional grid searching structure to minimize the overall minimum discrimination information (MDI)
distortion. In our method, no denoising filters are applied and there are no assumptions on any particular
noise characteristics.

We briefly illustrate the robust HMGMVQ using an example of Figure 1. Figure 1 (a) is clear gray
scale aerial image of size $512 \times 512$ ranged from 0 to 255. We add 10% of Salt and Pepper noise to
the clean image in the sense that we replace 10% of total $512 \times 512$ pixels with salt "0" or pepper
"255" randomly. This corrupted image is shown in Figure 1 (b). The hand labelled true class is shown
in Figure 1 (c). HMGMM segmented result is shown in Figure 1 (d), which shows poor performance of
HMGM for noisy input.

The poor performance of the basic HMGMVQ is from the covariance mismatch between the GMVQ
codebook using noiseless images and noisy testing images. To see this in detail, the matrix $\hat{\Sigma}$ shown
below is the one of the covariance matrices in the trained GMVQ codebook used to classify Figure 1
(a). The second matrix $\hat{\Sigma}_{\text{noisy}}$ is the estimated covariance matrix estimate for the noisy image Figure 1
(b) using the proposed method. The $\hat{\Sigma}_{\text{noisy}}$ enlarges the diagonals and shrinking the off-diagonals of $\hat{\Sigma}$.
The proposed algorithm adjusts the covariance matrix estimates $\hat{\Sigma}$ in GMVQ so that they are consistent to the covariance matrices of testing images with noise. With these adjusted covariance matrix estimates, the HMGMM procedure could be used to segment the noisy image Figure 1 (b). The specific covariance matrix adjustment proposed takes the form

$$\hat{\Sigma}_{\text{adj}} = \lambda_1 \text{diag}(\hat{\Sigma}) + \lambda_2 \left( \hat{\Sigma} - \text{diag}(\hat{\Sigma}) \right),$$

for an appropriately chosen $\lambda_1$ and $\lambda_2$. Two parameters $\lambda_1$ and $\lambda_2$ as multiplication factors of diagonal and off-diagonal elements in the covariance matrix were found by the MDI minimization framework. We sample the two dimensional $(\lambda_1, \lambda_2)$ domains and calculate the total MDI between the adjusted GMVQ with (1) and the testing image. We claim that two parameters that minimize MDI provide code books that are a good match to the corrupted image. We then show that the modified GMVQ indeed performs much better than the direct application of the basic GMVQ (which is designed for noiseless images) to noisy images in terms of the probability of error and visual segmentation results. The proposed method does not require retraining of the codebook, but simply adjusts the covariance matrices used in the GMVQ codewords. In view of image restoration, the proposed method is different from traditional methods because we are not interested in the restoration of the original image, but are instead interested in the segmentation of the hidden class while the observed image is still corrupted by noise. The traditional approaches restore noisy images to get clean images as front-end processing for segmentation. For S&P noise and independent Gaussian additive noise, median filtering, and blind de-convolution are commonly used respectively.

To illustrate our procedure, we focus on two types of noise, Salt and Pepper (S&P) noise and independent Gaussian additive noise. First, S&P noise has wide usage in image degradation. In S&P noise, only a few pixels are noisy and they are very noisy, while others are not tainted. This is different from Gaussian noise, which can alter all pixel values. Images corrupted with S&P noise look as if someone had sprinkled white and black dots across images. S&P noise occurs often in the case of overflow and
underflow of a noisy channel, camera and scanner sensor malfunctioning, or memory errors in hardware.
S&P noise can be considered as a type of impulse noise, which has minimum and maximum dynamic range. For example, if the channel is binary symmetric channel with certain cross-over probability, where this is the probability that each bit can be flipped, then the pixels will appear as black or white dots as the most significant bits (MSB) are flipped. This is distinctive since the mean square error (MSE) contribution from MSB change is at least three times that of other bits. Due to the non-linearity involved with impulse noise, non-linear filtering [3] based methods are a popular remedy to de-noise the image. Median filtering is popular since it has proven to have good denoising performance and is computationally efficient. However, it tends to smear the edges and details if the noise level is higher than 50%. Many improvements have been reported in the literature, including adaptive median filters, multi-state median filters, and median filters based on homogeneity information [3], [4]. In most cases, the noisy pixel has to be replaced by the median values and thus the details and edges are altered.

On the other hand, Gaussian additive noise is a widely used model for various real world noises. The Gaussian corrupted noisy results have blurry edges and reduced contrast. In the case of Gaussian noise, all the pixels are corrupted due to the heavy tail of Gaussian, thus makes the de-noising harder. A combination of least square based methods with an edge-preserving regularization function [35] non-smooth data fidelity terms ($l_1$) was proposed for the restoration of Gaussian noised images [26].

The remainder of the paper is organized as follows. In Section 2, we show in detail how the covariance matrix estimate is biased when test images are suffered from noises, particularly for the case of the Salt and Pepper noise and independent Gaussian noise. Thus, it provides a good theoretical justification to introduce the form (1) of adjustment. In Section 3, we introduce a modified HMGMM based segmentation procedure with the adjusted covariance matrix estimate. We further introduce the procedure to choose tuning parameters $\lambda_1$ and $\lambda_2$. In Section 4, we apply the adjustment procedure to segment aerial images suffering from noise. We compare the performance of our method with HMGMM and a median filter based method. Test images include S&P noise and independent Gaussian noise with various levels of intensities. The results show that the modified procedures on noised images perform as good as the HMGMM with noiseless images. Section 5 concludes the paper.

II. NOISE AND COVARIANCE ESTIMATE

The HMGMM is a context-based classifier. It divides the full image into many sub-blocks (for example, $8 \times 8$ sub-blocks) and classifies them using distance or distortion measures such as Kullback’s minimum discrimination information (MDI) [21]. To compute the MDI distortion between a sub-block and a
codeword or template, the mean vector and covariance matrix of a 2 × 2 unit within a sub-block is estimated and compared with the covariances associated with the codewords or templates.

In this section, we consider the effects of S&P noise on the estimates of mean vectors and covariance matrices in GMVQ-based classifiers. The covariance estimates obtained by GMVQ classifiers are consistent estimates of the true covariance of the observed images. Below we see how the true covariance of observed images with noise varies from that of clean images.

Let \((Y_s, Y_t)\) be any two adjacent observed values in a 2 × 2 unit where \(s\) and \(t\) are indices of any 2 × 2 unit within an 8 × 8 sub-block. They could be horizontally, vertically, or diagonally adjacent to each other. Let \((X_s, X_t)\) be the clean image without noise corresponding to \((Y_s, Y_t)\). We assume the clean image \((X_s, X_t)\) has mean 0 without loss of generality. Let \(Z_s\) and \(Z_t\) be binary variables indicating whether \(Y_s\) and \(Y_t\) are noised or not, respectively. For independent (Gaussian) noise, we assume that \(Z_{t,s}\) are IID random variables from a Bernoulli distribution with probability \(\alpha\). Thus, \(\alpha \times 100\%\) of pixels in a testing image are contaminated by noise. To be specific,

\[
Y_t = X_t + Z_t \epsilon_t
\]  

For S&P noise, we assume that

\[
Y_t = X_t (1 - |Z_t|) + c Z_t
\]  

where \(Pr(Z_t = 1) = Pr(Z_t = -1) = \alpha\), \(Pr(Z_t = 0) = 1 - 2\alpha\) where \(Z_t \in \{-1, 0, 1\}\), and \(c = 255\) for an 8 bits per pixel (bpp) image. Thus, the mean vector of added noise is 0.

When an image is contaminated by single pixel S&P noise, the covariance matrix of the noisy image has smaller elements in both diagonal and off-diagonal elements than the one of the clean image has. To see this in detail,

\[
\text{var}(Y_t) = \text{var}\left\{ E(Y_t | Z_t) \right\} + E\{ \text{var}(Y_t | Z_t) \} = (1 - 2\alpha)\text{var}(X_t) \leq \text{var}(X_t)
\]  

and

\[
\left| \text{cov}(Y_s, Y_t) \right| = \left| E(Y_s Y_t) \right| = \left| E\{ E(Y_s Y_t | Z_s, Z_t) \} \right| = (1 - \alpha)^2 \left| E(X_s X_t) \right| \leq \left| \text{cov}(X_s, X_t) \right|
\]

Thus, a good adjustment to the single pixel S&P noise is to shrink both diagonal and off-diagonal elements.
In the Gaussian case, \( \text{cov}(Y_s, Y_t) \) is not changed for \( s \neq t \). If independent and identically distributed Gaussian noise is added, then
\[
\text{var}(Y_t) = \text{var}(X_t) + \sigma^2
\]
Thus, an appropriate adjustment of covariance matrix estimate of \( X_t \) obtained from observation \( Y_t \) is to shrink diagonal elements and to keep off-diagonal elements of computed covariance matrix estimate.

As shown in the above two examples, the magnitude and the type bias in covariance estimates strongly depend on the type of noise. This observation is generally true for any type of noise. Since it is difficult to infer particular noise characteristics from observed images, we propose a general procedure which works well regardless of the specific type of noise.

III. TWO DIMENSIONAL GRID SEARCHING CRITERIA FOR COVARIANCE MODIFICATION

In this section, we introduce a procedure to modify the covariance matrices of GMVQ so as to reduce the bias caused by noise. By doing so, we can use existing GMVQ-based classifiers to segment images with covariance matrices perturbed from those of the training set. We propose to modify the covariance matrices as in (1) for appropriately chosen \( \lambda_1 \) and \( \lambda_2 \):
\[
\hat{\Sigma}_{\text{adj}} = \lambda_1 \text{diag}(\hat{\Sigma}) + \lambda_2 \left( \hat{\Sigma} - \text{diag}(\hat{\Sigma}) \right),
\]
(7)
As stated in the previous section, the covariance matrices of GMVQ are consistent estimates of the covariance matrices of the clean image, but are not consistent with the noisy test image. In the previous section, we demonstrated that S&P noise shrinks the covariance matrices of clean images. Thus, we can weaken this bias by modifying the covariance matrices of GMVQ. Since the type and magnitude of noise which contaminates the image is not known, we provide a procedure to choose the tuning parameters \( \lambda_1 \) and \( \lambda_2 \) from the noisy images.

The HMGMM constructs a GMVQ code book from a given image database. Let \( f_{ij} \), for \( i = 1, 2, \ldots, k \) and \( j = 1, 2, \ldots, n_i \), be the elements in the code book, which are multivariate normal distributions (or the covariance matrices which define the distributions) in our example, where \( i \) is the index of the class and \( j \) is that of the mixture components. In addition, let \( f_{ij}(\lambda) \) be the modification of \( f_{ij} \) with given \( \lambda = (\lambda_1, \lambda_2) \). Thus \( f_{ij}(\lambda) \) is a multivariate normal distribution having a modified covariance matrix with given \( \lambda \).

We choose \( \lambda = (\lambda_1, \lambda_2) \) to minimize the sum of the above MDI distances over all sub-blocks. Let \( h_b \) be the estimated Gaussian distribution of \( b \)-th sub-block. Given a particular distortion measure, \( h_b \)
with an optimally selected $\lambda$ is closest to a single Gaussian mixture component in a modified code book. Thus, we choose $\lambda$ to minimize

$$C(\lambda) = \sum_b \min \left\{ \text{MDI}(h_b, f_{ij}(\lambda)) \right\}$$

(8)

where $i$ is the HMGMM class, $j = 1, 2, \ldots, n_i$ and the summation is over all blocks of the testing image.

In summary, the overall covariance adjustment procedure is as follows:

1) Design GMVQ code books using images in the image database.
2) Find $\lambda_1$ and $\lambda_2$ to minimize $C(\lambda)$ (8).
3) For given $\lambda_1$ and $\lambda_2$, run HMGMM with the adjusted covariance estimates to segment new testing images.

IV. EXPERIMENTAL RESULTS

In this section, we describe a series of experiments to measure the performance of the covariance adjustment method. We consider two aerial images from the image suite used in HMGMM [27]: image 1 and image 11. These two images have very complex boundaries and mixtures between man-made and natural classes and hence provide challenge to segmentation, even without noise. We mix these images with S&P noise of 3% and 10% and Gaussian noise with variance 50 and 100. After mixing images with noise, we compare our method with HMGMM, and noise removal filters followed by HMGMM. In particular, we compare our performance to the ones obtained using a two-step procedure using a median filter-based method [4], [13] to clean S&P noise and blind de-convolution-based methods [13], [25] to clean Gaussian noise. In both cases HMGMM follows the denoising.

A. Covariance grid searching based on total MDI calculation

Based on (8), we search the best value of $\lambda$ to minimize the total MDI by sampling the two-dimensional grid space of $\lambda = (\lambda_1, \lambda_2)$. For the examples shown in this paper, the same method was applied to find $\lambda$. Table I reports the total MDI in sampled two-dimensional $\lambda$ space of image 11 corrupted by the S&P noise of 10% density. The choice of $\lambda_1=1.2$ and $\lambda_2=1.1$ gives the minimum total MDI distance of 3960. Thus, we modify the covariance matrices of GMVQ as

$$\hat{\Sigma}^{\text{adj}} = 1.2 \times \text{diag}(\hat{\Sigma}) + 1.1 \times (\hat{\Sigma} - \text{diag}(\hat{\Sigma}))$$

(9)

This is equivalent to modifying the GMVQ codebooks in the compressed domain without retraining. Our method changes covariance matrices in GMVQ based on the test image’s local statistics on the
Some combinations of $\lambda_1$ and $\lambda_2$ make the modified covariance matrix non-invertible (N/A in the table). As we can see in Table I, these non-invertible matrices constitute the lower triangular part. This is intuitively correct since this lower triangular part makes off-diagonals relatively bigger than diagonals, where the added noise already made the off-diagonals of the covariance matrix large, thus the matrix becomes closer to a singular matrix. The modified covariance gives a performance error rate of 12.6\% and is shown in Fig. 5 (d). In the table I, $\lambda_1=1.0$ and $\lambda_2=1.0$ correspond to an unmodified covariance matrix, which produces a total MDI of 7399 and an error rate of 48.1\% with the segmentation result in Fig 5 (c). We can visually verify how substantial the improvement is with the modified covariance matrix in GMVQ codebook; the reduction of total MDI is from 7399 to 3960 and that of the error rate is from 48.1\% to 12.6\%.

Table II shows other results of (8). These results are for image 1 corrupted by independent Gaussian noise with variance 50. When Gaussian noise is added to the image, the final image needs to be scaled to fit 8 bpp range since Gaussian noise has infinite range. As in table I, the lower triangular parts make the modified covariance matrix non-invertible. The minimum MDI of 4147 happens to be with $\lambda_1=1.2$ and $\lambda_2=1.1$. This is a smaller MDI value than MDI value of 4614, produced with the unmodified covariance matrix with $\lambda_1=1.0$ and $\lambda_2=1.0$. According to (6), only diagonals should change for independent Gaussian noise so that $\lambda_1$ should change while $\lambda_2$ should be 1.0 for minimum MDI. This was not true in our experiment since we scaled the noisy image when we added Gaussian noise to the image. The unmodified covariance produces an error rate of 30.8\% and is shown in Fig. 12 (c). The modified covariance gives a performance error rate of 20.7\% and is shown in Fig. 12 (d). This shows the visual closeness to the desired segmentation result in Fig. 12 (e).
TABLE II

<table>
<thead>
<tr>
<th>$\lambda_1 \rightarrow$</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>5261</td>
<td>6016</td>
<td>5925</td>
<td>4965</td>
<td>5766</td>
</tr>
<tr>
<td>0.9</td>
<td>N/A</td>
<td>4492</td>
<td>11232</td>
<td>5205</td>
<td>6486</td>
</tr>
<tr>
<td>1.0</td>
<td>N/A</td>
<td>N/A</td>
<td>4614</td>
<td>5096</td>
<td>4471</td>
</tr>
<tr>
<td>1.1</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>7564</td>
<td>4147</td>
</tr>
<tr>
<td>1.2</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>4388</td>
</tr>
</tbody>
</table>

**TOTAL MDI distance of grid search of $\lambda_1$ and $\lambda_2$ from 0.8 to 1.2. Test image 1 is corrupted by Gaussian noise $\mathcal{N}(0, 50)$. $\lambda_1=1.2$ and $\lambda_2=1.1$ gives the minimum MDI distance of 4147. N/A means that the covariance is non-invertible, thus the total MDI cannot be calculated.**

B. Segmentation of S&P Noised images

With S&P noise, we added image 1 and 11 with 3% and 10% noise density, respectively. The error rate results for our covariance adjustment methods are summarized in Table III. The table shows that our method has lower error rate than median filter does in both images in most cases we considered; with 10% noise density, our methods performs better than the median filter based method in both images; with 3% noise density, our method do better in image 1 but not in image 11. Also, unlike the median filter based method, the performance of proposed modified HMGMVQ method is stable in error rate around 20% regardless of noise levels and images. The visual segmentation results are shown in Fig. 4, Fig. 5, Fig. 6, Fig. 7, for S&P noise of 3% and 10% for images 11 and 1, respectively. It is clear that our covariance adjustment method produces visually similar segmentation output to the noiseless segmentation result in these examples.

We first cleaned the image with a median filter based method and then applied HMGMM to the cleaned image. The median filtered HMGMM segmentation results are shown in Fig. 2 and Fig. 3. The median filter based approach works relatively well with image 11 with error rates of 14.1% and 18.8% (see Fig. 2 (e) and (h)). It performs poorly, however, for image 1 with error rates of 32.5% and 33.3% (see Fig. 3 (e) and (h)). Visual segmentation results in Fig. 3 (e) and (h) show that most man-made structures are not recognized except for the football field neighborhood. We applied our covariance grid search algorithm to the four S&P noise contaminated images and compared the segmentation performance. For the case of S&P noised image 11 with 3% density, Fig. 4 (c) shows poor performance of direct application of HMGMM to the noisy image 11.

Fig. 4 (d) is the result of our method. It has a higher error rate compared to Median filtered image with HMGMM segmentation results in Fig. 4 (f), but the segmentation result of Fig. 4 (d) matches well
TABLE III

Comparison of misclassification error rate for aerial image 11 and image 1 between covariance adjustment method with HMGMM variants: N+HMGMM — the noisy image is segmented with HMGMM, our method — the noisy image is segmented with covariance modified HMGMM, NF+HMGMM — noise-free image is segmented with HMGMM, M3 × 3+HMGMM — noisy image is filtered through 3 × 3 median filter and then segmented with HMGMM.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>N+HMGMM</th>
<th>our method</th>
<th>NF+HMGMM</th>
<th>M3 × 3+HMGMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P(3%) with image 11</td>
<td>49.1</td>
<td>20.3</td>
<td>15.4</td>
<td>14.1</td>
</tr>
<tr>
<td>S&amp;P(10%) with image 11</td>
<td>48.1</td>
<td>12.6</td>
<td>15.4</td>
<td>18.8</td>
</tr>
<tr>
<td>S&amp;P(3%) with image 1</td>
<td>49.3</td>
<td>22.6</td>
<td>14.8</td>
<td>32.5</td>
</tr>
<tr>
<td>S&amp;P(10%) with image 1</td>
<td>49.7</td>
<td>19.7</td>
<td>14.8</td>
<td>33.3</td>
</tr>
</tbody>
</table>

visually to the result of Fig. 4 (e). Note that Fig. 4 (e) is the noiseless HMGMM segmentation result. For S&P noise of 10% density, we see that our covariance adjustment method in Fig. 5 (d) performs better than the median filter-based approach in Fig. 5 (f). Surprisingly, our result in Fig. 5 (d) shows a lower error rate than Fig. 5 (e), which is the noiseless image segmented with HMGMM. This means that the original HMGMM relies on the performance of GMVQ, designed with the generalized Lloyd algorithm [24]. Since the Lloyd algorithm is sensitive to the initial condition, this shows that it is possible that the grid searching of covariance matrix produces better codebooks than the original GMVQ does.

Fig. 6 (d) and Fig. 7 (d) demonstrate the performance of the covariance adjustment method for S&P noise at 3% and 10%, respectively. The corresponding error rates are 22.6% and 19.7%, respectively. The pictures are visually quite similar to the ideal (no noise) segmentation results of Fig. 6 (b) and Fig. 7 (b). Fig. 6 (c) and Fig. 7 (c) depict the segmentation results of the direct application of HMGMM to the noised image. The resulting error rates are significantly worse: 49.3% and 49.7%, respectively. When applying HMGMM to median filter-cleaned images, the results are still poor in that the segmentation results can not capture most of man-made structures with error rates of 32.5% and 33.3%; see Fig. 6 (f) and Fig. 7 (f), respectively.

C. Segmentation of Gaussian noised images

The four images from image 11 and 1 with added independent Gaussian noise with variance of 50 and 100 are shown in Fig 8 (c), (d) and in Fig 9 (c), (d). The error rate results for our covariance adjustment methods are summarized in Table IV, which shows that our method has a lower error rate than the direct application of HMGMM on the noisy images. The visual segmentation results are shown in Fig. 10, Fig. 11, Fig. 12, Fig. 13, for Gaussian noise of variance 50 and 100 for images 11 and 1, respectively.
Fig. 2. S&P noise image and median filtered image 11: (a) original 11, (b) hand labelled class 11, (c) image 11 with S&P noise 3%, (d) $3 \times 3$ Median filtered image of noisy image (c), (e) Segmentation of image (d) by HMGMM (error rate 14.1%), (f) image 11 with S&P noise 10%, (g) $3 \times 3$ Median filtered image of noisy image (f), (h) Segmentation of image (g) by HMGMM (error rate 18.8%). White: man-made; gray: natural.

The covariance adjustment method produces visually closer segmentation to the noiseless segmentation result than the direct application of HMGMM onto noised images does.

Several methods to restore Gaussian contaminated noise have been reported in the literature [5], [12], [33]. However, most methods such as constrained least squares filtering, Wiener filtering, or nonlinear adaptive filter based approaches require complete knowledge of the noise characteristics. Our method does not assume any noise characteristics. To be fair to our method, the comparison method should assume a minimum amount of information on the noise characteristics. One such methods is a blind de-convolution method, which does not rely on the specific knowledge of the point spread function (PSF). However, it still has to assume that the noise is Gaussian. Holmes et al. [18] and Hanisch et al. [17] applied a maximum likelihood estimator (MLE) blind de-convolution method to microscopy and astronomy. We used a blind de-convolution method with a Gaussian assumption to clean the noise and then segment the cleaned image using HMGMM. In all four cases of images 1 and 11 with Gaussian variance of 50
and 100, the algorithm fails and segments the images into a single class. This implies that a restoration technique such as the de-convolution method changes the covariance structure severely and hence requires re-training procedure.

The results of the covariance adjustment method are compared with direct application of HMGMM in Fig. 10 and Fig. 11 for image 11. Our results at Fig. 10 (d) and Fig. 11 (d) show performance with error rates of 17.9% and 17.6% and striking visual similarity to the HMGMM segmentation of noiseless images in Fig. 10 (e) and Fig. 11 (e). Noisy segmentation performance of the direct application of HMGMM becomes poor when the Gaussian noise power is increased from 50 to 100 (see Fig. 10 (c) and Fig. 11 (c)). For a busy image like image 1, our methods in Fig. 12 (d) and Fig. 13 (d) substantially outperform the performance of noisy segmentation with direct HMGMM application, contaminated with Gaussian noise power of 50 and 100, as shown in Fig. 12 (c) and Fig. 13 (c). In particular, for noise power of 100 for image 1, direct HMGMM applied to a noisy image performs very poorly with an error rate of
Fig. 4. 3% S&P noise segmentation results comparison for image 11: (a) Original image 11, (b) Hand labelled class, (c) N+HMGMM: error rate = 49.1%, (d) Our method: error rate = 20.3%, (e) NF+HMGMM: error rate = 15.4%, (f) M3 × 3+HMGMM: error rate = 14.1%. White: man-made; gray: natural.

Fig. 5. 10% S&P noise segmentation results comparison for image 11: (a) Original image 11, (b) Hand labelled class, (c) N+HMGMM: error rate = 48.1%, (d) Our method: error rate = 12.6%, (e) NF+HMGMM: error rate = 15.4%, (f) M3 × 3+HMGMM: error rate = 18.8%. White: man-made; gray: natural.
Fig. 6. 3% S&P noise segmentation results comparison for image 1: (a) Original image 1, (b) Hand labelled class, (c) N+HMGMM: error rate = 49.3%, (d) Our method: error rate = 22.6%, (e) NF+HMGMM: error rate = 14.8%, (f) M3 × 3+HMGMM: error rate = 32.5%. White: man-made; gray: natural.

Fig. 7. 10% S&P noise segmentation results comparison for image 1: (a) Original image 1, (b) Hand labelled class, (c) N+HMGMM: error rate = 49.7%, (d) Our method: error rate = 19.7%, (e) NF+HMGMM: error rate = 14.8%, (f) M3 × 3+HMGMM: error rate = 33.3%. White: man-made; gray: natural.
59.3% in Fig. 13 (c), while our method performs better with an error rate of 32.6% in Fig. 13 (d).

D. Segmentation of noiseless images

In the real world situation, it is hard to know whether the image to be segmented is noisy or not. To show that our MDI framework is a general robust solution to image segmentation problem, we applied the same exact procedure to the noiseless images. The result on noiseless image 1 is shown in Fig. 14 (a) with error rate 19.3%. This result was achieved from the same minimum MDI searching criteria in equation 8 with $\lambda_1 = 1.2$ and $\lambda_2 = 1.1$. Fig. 14 (b) is the result on noiseless image 11 with error rate 18.5%. This was obtained with $\lambda_1 = 0.8$ and $\lambda_2 = 0.8$. These results are close to Fig. 6 (e) and Fig. 4 (e), which are noiseless segmentation results using HMGMM [27]. The error rates are a little bit higher, but the visual segmentation results are close to the HMGMM results. This shows that our algorithm is a robust framework and does not require the knowledge of noise presence for image segmentation.

V. CONCLUSIONS

We formulated segmentation of a noisy source as a quantization system for a noisy source using an MDI distortion measure with a revised covariance estimation method based on the “shrinking” property of covariance estimates in the presence of noise. A grid searching algorithm was applied to modify HMGMVQ in order to obtain quantizers which operate on the noisy source, while producing a good reproduction of the clean source in the sense of minimizing the average MDI distortion. The approach
Fig. 9. Gaussian noise corrupted image 1: (a) Original image 1, (b) Hand-labelled class 1, (c) Gaussian N(0,50) corrupted image, (d) Gaussian N(0,100) corrupted image. White: man-made; gray: natural.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>N+HMGMM</th>
<th>Our method</th>
<th>NF+HMGMM</th>
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<tbody>
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<td>N(0,50) with image 11</td>
<td>23.1</td>
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<td>15.4</td>
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<tr>
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<td>N(0,100) with image 1</td>
<td>59.3</td>
<td>32.6</td>
<td>14.8</td>
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</table>

**TABLE IV**

Comparison of the classification performance for aerial image 11 and image 1 between covariance adjustment method with HMGMM variants: N+HMGMM means that the noised image is segmented with HMGMM. Our method means that the noisy image is segmented with a covariance adjustment modified HMGMM. NF+HMGMM means that the noise-free image is segmented with HMGMM.

was shown to significantly improve the performance of HMGMM-based segmentation in the presence of noise and to provide better performance than two popular denoising techniques applied in a two step approach to segmentation where the noisy image is first denoised and then processed using a segmentation algorithm designed for the clean source. The proposed algorithm is simple in that retraining of codebooks is not required and prior distributions of image noise do not need to be known.

**REFERENCES**


Fig. 10. $N(0,50)$ Gaussian noise segmentation results comparison for image 11: (a) Original image 11, (b) Hand labelled class, (c) N+HMGMM: error rate = 23.1%, (d) Our method: error rate = 17.9%, (e) NF+HMGMM: error rate = 15.4%. White: man-made; gray: natural.

Fig. 11. $N(0,100)$ Gaussian noise segmentation results comparison for image 11: (a) Original image 11, (b) Hand labelled class, (c) N+HMGMM: error rate = 36.3%, (d) Our method: error rate = 17.6%, (e) NF+HMGMM: error rate = 15.4%. White: man-made; gray: natural.
Fig. 12. N(0,50) Gaussian noise segmentation results comparison for image 1: (a) Original image 1, (b) Hand labelled class, (c) N+HMGMM: error rate = 30.8%, (d) Our method: error rate = 20.7%, (e) NF+HMGMM: error rate = 14.8%. White: man-made; gray: natural.

Fig. 13. N(0,100) Gaussian noise segmentation results comparison for image 1: (a) Original image 1, (b) Hand labelled class, (c) N+HMGMM: error rate = 59.3%, (d) Our method: error rate = 32.6%, (e) NF+HMGMM: error rate = 14.8%. White: man-made; gray: natural.
Fig. 14. Noiseless image segmentation results on: (a) image 1, (b) image 11. White: man-made; gray: natural.


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